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Le projet avait pour but d'expérimenter un module de mathématiques spécialement conçu par le conseiller Don Reid pour apprendre à résoudre des problèmes. Un guide pédagogique a été fourni aux enseignants de 5e année, ainsi qu'un entraînement ad hoc. A la fin de la période d'expérimentation, 12 semaines, on a questionné les enseignants sur leur attitude et celle des élèves envers la méthode proposée et on a administré divers tests de mathématiques aux élèves. Les résultats indiquent que le module a été bien accueilli et utilisé efficacement, mais il semble que le temps consacré à l'expérimentation en classe n'a pas été assez long.

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DEVELOPMENT, IMPLEMENTATION AND EVALUATION

OF A MODULE DESIGNED TO IMPROVE PROBLEM SOLVING SKILLS

Development, Implementation and Evaluation

of

A Module Design to Improve Problem Solving Skills

Grade 5

Ellen Adiv

Acknowledgements

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Résumé

Le but de ce projet était d'introduire dans la classe un outil pédagogique visant le développement des habiletés nécessaires pour résoudre les problèmes en mathématiques.

La réalisation du projet s'est faite en quatre étapes: (1) élaboration d'un guide pédagogique, (2) entraînement des professeurs, (3) expérimentation dans la classe et (4) évaluation.

L'expérimentation s'est faite au niveau de la 5^{e} année pendant une période de douze semaines. Pour évaluer l'efficacité de l'outil,on a administré plusieurs pré-tests et post-tests en mathématiques. Un groupe témoin était inclus pour but de comparaison. A la fin de la période d'expérimentation, on a demandé aux professeurs de remplir un questionnaire qui traitait des attitudes des élèves et des professeurs envers l'outil pedagogique.

Les résultats de l'évaluation indiquent que: (1) les réactions des professeurs et des élèves envers l'outil pédagogique ont été très positives, (2) l'entraînement des

professeurs a été satisfaisante, (3) le taux de participation des élèves aux travaux de classe a été très élevé, (4) à un des tests en mathématiques, les résultats obtenus au post-test étaient nettement supérieurs à ceux obtenus au pré-test et la performance du groupe expérimental s'est améliorée plus que celle du groupe témoin, (5) aux autres tests, la différence entre les résultats du pré-test et ceux du post-test n'était que très légèrement ou pas du tout significative.

Les résultats de cette étude nous mènent à conclure que l'effet de l'outil pédagogique sur le développement des habiletés de résolution de problèmes a été positif mais restreint. Ce résultat est attribué au temps limité qui a été consacré à l'expérimentation de l'outil pédagogique dans la classe.

Summary

The purpose of this project was to develop, implement and evaluate a module designed to improve problem solving skills.

The project consisted of four stages: (1) development of a pedagogical guide, (2) training of the teachers, (3) implementation of the module in the classroom and (4) evaluation.

The module was implemented in grade 5 during a twelve week period. Several pre- and post-tests in mathematics were administered to assess the effectiveness of the module. A control group was included for comparison purposes. A questionnaire designed to elicit information about teacher and student attitudes to the module was administered to the teachers at the end of the implementation period.

The results of the evaluation indicate that (1) teacher and student reactions to the module were extremely positive, (2) teacher training was reported to have been adequate, (3) student involvement during the classroom sessions was very high, (4) student performance on one of the tests improved significantly between the pre-test and the post-test and the gains made by the experimental group were more

pronounced than those made by the control group. (5) Differences between the pre-test and post-test scores on the other tests were either not significant or marginally significant.

The findings lead to the conclusion that the module had a positive though limited effect on the development of the students' problem solving skills. The outcomes are attributed primarily to the relatively short span of time during which the module was implemented in the classroom.

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Problem solving occurs in many different situations. In fact, it can be argued that every time people find themselves in a situation in which they see no obvious means to achieve their goal, they have to engage in problem solving.

Within the context of a formal classroom setting, problem solving has always been part of the mathematics curriculum. However, how best to teach the skills related to problem solving has been a matter of continuing concern to educators.

Problem Solving - The State of the Art

The emphasis given to problem solving in the eighties (see, for instance, Branca, 1980; Fennell, 1981; Charles and Lester, 1982; Avital, 1983; Lester, 1983; Chisko, 1985) reflects the growing awareness that neither the behavioristic approach to the learning of mathematics which dominated from the thirties to the fifties, nor the much broader "Modern Mathematics" approach which emphasized content over drill, provided students with effective tools for problem solving. Indeed, a large-scale study of achievement in mathematics conducted in the United States during the 1977-1978 school year revealed that problem solving abilities had declined during the seventies (Carpenter et al., 1980). It is partially as a result of these findings that mathematics

educators have concentrated on devising new approaches to the teaching of problem solving skills.

New Instructional Approaches

One of the most dramatic changes that has taken place in reassessing instructional approaches to problem solving is the shift in focus from a product oriented approach to a process oriented approach. In other words, whereas the teaching of problem solving traditionally emphasized the final solution or answer to the problem as its goal, modern instructional guidelines give priority to the intermediate stages where the learner must devise a plan how to solve the problem (Schoenfeld, 1980; Kantowski, 1981).

This shift in emphasis from product to process has several implications. First of all, it necessitates the identification of the steps involved in arriving at the solution of a problem. Polya's (1957) model is perhaps the most widely adapted model for this purpose. According to Polya, problem solving involves four basic steps

- understanding the problem
- devising a plan (for its solution)
- carrying out the plan
- checking the answer

This four-step model is not new. Unfortunately, all too often the scope accorded to each step, particularly steps one and two is much too narrow. Understanding, for instance, is frequently limited to simply comprehending the written text. Carpenter et al. (1980) found that many students read a problem and immediately attempt to calculate the answer by using the given numbers in a simple mathematical operation. Lester (1975) noted that students often misread and misinterpret problems, thus, failing to coordinate the multiple conditions that make up the problem. As Polya (1957) sees it, understanding involves grasping the entire network of relationships that holds between the factors stated in the text.

Similarly, Polya's second step, namely planning how to solve the problem, is frequently equated with merely choosing a mathematical operation, at least at the elementary level. This oversimplification may be due, to some extent, to the nature of the problems which are presented in most mathematics books (Carpenter et al. 1981). Indeed, the findings of the second National Assessment of Educational Progress (reported in Carpenter et al.) clearly show that students are markedly more successful at solving routine one-step problems than problems that involve some kind of deeper analysis. Kantowski (1981) claims that these one-step problems are little more than a number exercise

stated in verbal form. Even when the problems are of a more complex nature, the instructional procedure followed in presenting these problems is essentially the same. In most cases, the teacher gives a sample problem and explains the steps to be followed in order to arrive at the solution. This sample problem then becomes a model which the students use to solve other similar problems. Dessant (1981) argues that although this procedure provides learners with some guidance, it is primarily a "matching-analysis procedure" which does not motivate the learner to think the problem through (p.11).

Today mathematics educators propose that students be exposed to problems for which they have no readily applicable model. To find the solution to this type of problem, learners have to have recourse to a variety of discovery techniques some of which are not necessarily related to mathematics. According to Schoenfeld (1980) such discovery techniques involve heuristic processes which can vary from one individual to another. The central function of heuristics is to help problem solvers "approach, understand and/or marshall their ressources to solve problems." (p.432).

The introduction of heuristics into the classroom is still in its infancy. The task is a difficult one since its key

component involves the coordination between prior experience, creativity and an in-depth understanding of the problem that is to be solved. In order to encourage students to apply heuristic procedures to problem solving, Charles and Lester (1982) propose to teach students a number of strategies starting as early as grade 3. Most of the strategies provide the learner with a visual representation of the relationship that holds between the factors given in the problem, e.g.,draw a picture or find a pattern. Others appeal to logic (use logical reasoning) and to common sense, e.g., guess and check. In addition, the authors state several conditions which they think will help students become better problem solvers. First, the problem must be sufficiently interesting to the students so that they will want to solve it. Second, the problem must be appropriate to the students' prior experience in mathematics, to their reading ability and to their analytical ability. Finally, affective factors such as tolerance for ambiguity, perseverance, and the level of anxiety the learner experiences also influence success in problem solving. Their advice to teachers is to devote a serious amount of time to problem solving distinct from other instructional time assigned to mathematics. They also put much emphasis on a positive classroom climate, one which is challenging without being frustrating, a setting where the

emphasis is on the search for a solution rather than on getting the correct answer.

Other guidelines for a successful problem solving program suggest that students be encouraged to discuss the problem and suggest ways to solve it before they actually work out the answer (Lester, 1983). Moreover, in order to show a student that problems can be solved in more than one way, teachers should reward risk takers and accept unusual solutions.

To meet the challenges of this new instructional approach, two basic needs have to be met: teachers must be adequately prepared for their task and appropriate teaching materials must be developed. Most of the textbooks currently in use put little emphasis on how to actually teach problem solving. Wearne (1981) notes:

> "Redesigning textbooks so they develop understanding and involve students in problem solving activities is more than a cosmetic change of including a few additional changes in a book." (p.40)

The teaching module that is described in this project was developed with these issues in mind.

The Project

The Problem

The difficulty school children experience with problem solving has also been noted in Canada (see, for instance, Avital and Shettleworth, 1968; Ross and Maynes, 1982; Sherrill, 1983).

The issue is particularly acute in Inner City schools where low achievement in mathematics is wide-spread. An analysis of the 1984 results of the mathematics tests administered by the Protestant School Board of Greater Montreal (PSBGM) in its schools reveals that approximately three fourth or 75% of the items pertaining to problem solving were answered incorrectly by students in Inner City schools. Further insights into the performance of students in the PSBGM's Inner City schools can be gained from a recent study by Adiv and Doré (1985) who examined student achievement in relationship to the socio-economic back ground of students in both Inner and non-Inner City schools. The findings of the study indicate that students from socio-economically disadvantaged homes scored consistently lower on problem solving tasks than students from less disadvantaged homes who attended the same classes.

Since the majority of the students in Inner City schools come from socio-economically disadvantaged homes, it was decided that there was a real need in these schools for some kind of program that would focus specifically on problem solving.

Purpose of the Project

The purpose of this project was to design, implement and evaluate a module designed to improve problem solving skills. In keeping with the present-day notions on how to help students develop better problem solving skills, it was decided to focus on process problems and to use an instructional approach that would expose the learner to some of the problem solving strategies discussed by Charles and Lester (1982). The module was designed for grade 5 students. It was felt that the introduction of the module at this grade level would ensure that the students were cognitively mature enough to handle the material. On the other hand, it would give them enough time to improve their problem solving skills before they complete their elementary school studies.

The project consisted of four parts: (1) the development of a pedagogical guide, (2) the training of the teachers, (3) the implementation of the module in the classroom and (4) the evaluation. Each of these parts is discussed below.

(1) The Pedagogical Guide

The pedagogical guide was developed by Don Reid, Mathematics Consultant at the PSBGM. It provides the teacher with a short introduction to the issues on teaching problem solving in the classroom and then goes on to present the following seven strategies proposed by Charles and Lester (1982):

(1) Draw a picture

(2) Make a table

(3) Make an organized list

(4) Ask intermediate questions

(5) Guess and check

(6) Find a pattern

(7) Use logical reasoning

Each strategy is explained by means of one or two examples and is followed by several problems which can be solved by applying that particular strategy.

Furthermore, Mr. Reid suggests using Polya's four step model (1965) whenever possible. He also recommends a number of additional guidelines such as asking students questions and having them explain how they derived their solutions.

On the last page of the guide, the teacher is provided with a number of references for resource material.

It should be noted that the guide was designed specifically for the workshop and is not a handbook for use in the classroom. Its purpose is to enable teachers to look at problem solving in novel ways.

(2) Training of the Teachers

In order to prepare teachers to use a strategy oriented instructional approach, a three-day workshop was given at the end of January 1986, a short time prior to the time the module was to be introduced into the classroom. Each day's session was divided into two parts: a formal work period and an informal work period. The entire workshop was organized by Don Reid who also led the formal sessions. Using the pedagogical guide as a base, Mr. Reid briefly introduced the teachers to the notion of process problems and then proceeded to discuss the strategies, letting the teachers work out for themselves the additional problems given in the guide.

In addition to the pedagogical guide, the teachers were provided with a variety of problem solving resource materials

for use in the classroom and additional reference reading to broaden their scope of knowledge on the topic. The informal afternoon sessions were used to examine these materials, hold small group discussions and plan how to introduce this new approach to problem solving into the classroom.

By the end of the first day it was already clear that the workshop was a success. The teachers were fully aware that their students were not acquiring enough problem solving skills and were eager to try a new approach. Moreover, they found their actual participation in the problem solving activities stimulating.

Part of each day's formal session was devoted to questions that arose out of the work the teachers had done on their own during the previous day.

Mr. Reid and Professor Earlwanger from Concordia University, who attended the entire workshop, also made a number of suggestions pertaining to procedures teachers should use in the classroom. In particular, it was noted that the sessions should not be labeled as problem solving lessons and that they should be organized so as to actively involve the students in discussions.

With respect to the actual teaching of the strategies, two approaches were presented. Teachers could either work on many problems using the same strategy for a certain time and then go on to the next strategy (matched approach) or they could use different types of problems right from the beginning and each time apply the strategy that fits best to a given problem (unmatched approach). Other procedures pertaining to the implementation of the module such as classroom activities, choice of materials and time allocations were left open-ended so as to give each teacher maximum flexibility to accommodate the module into her lesson plan.

Two follow-up activities were undertaken in order to see how teachers were implementing the module in the classroom and to provide further guidance if necessary. The first was a series of classroom observations and the second was a one-day workshop given mid-way through the twelve-week period during which the module was being implemented. The workshop also provided an opportunity for the teachers to share their experiences and to discuss with Mr. Reid questions which had arisen in the course of the six weeks during which they had used the module.

(3) Implementation of the Module in the Classroom

Although each of the teachers introduced the module in her own way, there were a number of common features which characterized the learning experience these teachers created for their students. First of all, as suggested, they did not refer to them as problem solving sessions. Instead, labels such as "Brain Teasing" and "Brainstorming" were mostly used. These labels were very representative of what was happening in the classroom. Mr. Reid noted during his visits that a teacher would present a problem on the board and then a vivid discussion would follow to see how the problem could best be solved. If the teacher was using the problem to introduce a new strategy, he would usually lead the discussion and then demonstrate the strategy on the board. On the other hand, if the problem required the application of a strategy which had already been thought, she would let the students suggest as many ways of solving the problem as they could think of. The teacher would then let some of the children demonstrate their solutions on the board and discuss with them the different strategies that had been put into use.

Often a teacher would let the students do their brainstorming in small groups and would then discuss with them both the correct and the incorrect solutions. In this way, students were led to examine the reasons that caused certain solutions to be wrong.

Most teachers introduced the strategies one at a time using, in the initial stages, only problems related to that particular strategy (matched approach). However, as soon as the students had been familiarized with several strategies, teachers found it more challenging to give problems which required the use of different strategies (unmatched approach).

The students' reaction to this new instructional approach was extremely positive. Mr. Reid found that student involvement was very high in all the experimental classes. With respect to the children's use of the various strategies, Mr. Reid indicated that students seemed to apply the "draw a picture" strategy more frequently than the others strategies, even in cases where the problems were obviously related to a different strategy, one which had just been presented. It would seem that for these young students, the visual input provided by a picture is more meaningful than the input presented by other means even when the latter also provide a visual representation as in the case of a table or an organized list.

The feedback provided by the teachers during the second workshop showed that they felt very comfortable using the strategy approach and that the materials that had been provided

during the first workshop had been put to good use. Some teachers noted that their entire perception of how to teach problem solving had changed. However, concern was also voiced as to whether students would apply the problem solving strategies in other contexts, particulary in mathematics. It is precisely for this reason that an assessment of the students' achievement in this subject was included in the project.

(4) Evaluation

In order to assess the effectiveness of the module it was decided to measure several aspects of student performance in mathematics before and after the implementation of the module in the classroom and to compare, at each stage, the performance of these experimental students to that of a control group who was taught the regular mathematics program. Details about the student sample, the instruments and the administration procedures are given below.

The Sample

Two hundred and four students participated in the evaluation. One hundred of these students were experimental students and the remainder constituted the control group. The students came from seven different Inner City Schools. All the schools

belonged to the English sector of the PSBGM. One of the schools had two grade 5 classes. One of these classes belonged to the experimental group and the other to the control group. The other six schools had only one or one and a half classes at the grade 5 level. Thus, the remaining control students attended different schools than the experimental students.

The students came from many different socio-linguistic and ethnic backgrounds. Since this feature is a characteristic of most of the PSBGM's schools, it was impossible to hold this variable constant. However, the distribution of the different ethnic groups in the experimental classes was similar to that in the control classes. Moreover, most of the students included in the study had attended an English medium school from kindergarten or grade 1 and all spoke English fluently.

The Instruments

The following tests were administred to both the experimental and the control students: (i) a written mathematics test, (ii) an oral problem solving task; (iii) a reading test and (iv) a non-verbal test. In addition, a questionnaire was administered to the teachers who had implemented the module in their classrooms. Each of these instruments is briefly described below.

(i) The Written Mathematics Test

In order to see whether the new instructional approach had an effect on the students ability to solve problems as well as on their internalization of mathematical concepts, two kinds of mathematics tests were administered: a problem solving test and a mathematics concept test. The test battery selected for this purpose was the Canadian Tests of Basic Skills (CTBS), metric version level 11. The mathematics subtest pertaining to concepts contains thirty seven items and the problem solving subtest contains twenty-seven items. Twenty of the problems are straight foreward one or two-step product problems. The other seven problems are more process oriented. All the items on the two tests are multiple-choice items.

(ii) The Oral Problem Solving Task

The oral problem solving task was developed especially for this project. Its purpose is to try and discover what steps a student takes when he or she has to solve a problem. The student is given a problem in written form and is asked to solve it. A piece of scrap paper is provided for the purpose. After the student has given an answer to the problem, he or she is asked six follow-up questions. The first four questions elicit information about the procedures the student has followed in order to solve the problem. The last two questions ask how

difficult the student found the problem and whether he or she can think of another way of solving it.

(iii) The Reading Test

When problems are presented in written form, comprehension of the problem is heavily dependent on the student's ability to read the text in a meaningful way. Since this factor could influence the results of the mathematics tests that were used in this evaluation, it was decided to administer a reading comprehension test to all the students. Here again the test was selected from the CTBS battery, level 11. The test comprises nine texts of varying lengths. Each text is followed by several questions, (fifty-three in all). All the questions are followed by multiple-choice answers.

(iv) The Non-Verbal Test

In order to ensure that possible differences between the experimental group and the control group were not due to differences in mental capacity, the students were administered Raven's Standard Matrices test. This test, while not giving a direct measure of IQ, is designed to assess as accurately as possible a person's level of intellectual development. It is particularly suitable for testing children whose mother tongue is not English, since it involves no language skills.

(v) The Teachers' Questionnaire

Because of the experimental nature of the project, it was felt that feed-back from the teachers would provide valuable information regarding factors related to the actual teaching of the module. A questionnaire was, therefore, developed by the research team and handed to the teachers at the end of the twelve-week period. The questionnaire contained fifteen questions and covered four main topics: (1) details about the manner of implementation of the module, (2) adequacy of the training they had received, (3) adequacy of the amount of time allotted to teaching the strategies and (4) information about student reactions to the project. The teachers were also asked whether they had enjoyed teaching problem solving in this manner and whether they felt the project should have covered more ground. The questionnaire was anonymous to ensure confidentiality.

Administration Procedures

The two written mathematics tests and the oral problem solving task were administered twice, once prior to the introduction of the module into the classroom (pre-test) and once after the twelve-week period during which the module had been taught (post-test). Two different forms of the CTBS were used for this purpose (Form 5 for the pre-test and Form 6 for the post-test). The reading comprehension test and the non-verbal test were

administered only once.

All the tests, except the oral problem solving task, were administered to entire grade 5 classes in all the participating schools (experimental and control groups). However, the test results of students with sev re learning problems and those of non-native English speakers who had been in the country less than two years were later eliminated from the data.

The oral problem solving task was administered individually to approximately fifty oercent of the students (experimentals and controls ; for exact sample sizes, see Table 1). This test was also administered twice (pre-test and post-test). The problems presented during the pre-and post-tests were similar in nature but differed in terms of verbal content. The follow-up questions were left unchanged. The test was administered in a separate room and each session lasted approximately five minutes. The entire exchange between tester and student was tape-recorded.

Results

The students' scores on the reading test and on Raven's Standard Progressive Matrices test were analyzed first using a one-way analysis of variance procedure. The results pertaining to the reading test showed no significant differences between

the two groups ($F = 1.48$; $df = 1,201$). On the other hand, the analysis of the results on the Standard Progressive Matrices indicated that here the difference between the groups was significant ($F = 3,66$; $df = 1,199$; $p<.05$). Since this difference could influence the results of the mathematics tests, the latter were analyzed using a co-variance procedure (see the appendix for a brief explanation).

The written mathematics tests were submitted to a two-way analysis of co-variance with group (experimental versus control) and testing time (pre versus post) as the independent variables.

The data from the oral problem solving task did not lend itself to an analysis of variance and a χ^2 procedure was used here to determine whether differences between the groups were significant either during the pre-test or at the time of the post-test.

The findings of all of these analyses are summarized in Tables 2 and 3 and are briefly described below.

The Written Mathematics Test

On sub-test I of the mathematics test (mathematic concepts), there was a marginally significant increase in scores from the

test (experimentals, $\bar{X}$ = 20.47; controls, $\bar{X}$ = 20.33) to the post-test (experimentals, $\bar{X}$ = 21.07; controls, $\bar{X}$ = 20.74). On the other hand, there was no significant difference between the scores of the experimental group and those of the control group. Moreover, the Pre/Post x Group interaction was also not significant (see Table 2 for details).

On sub-test II of the mathematics test (problem solving), there was a substantial improvement in performance between the pre-test (experimentals, $\bar{X}$ = 15.40; controls, $\bar{X}$ = 15.38) and the post-test (experimentals, $\bar{X}$ = 17.00, controls, $\bar{X}$ = 16.00) and there was again no significant main effect for group. However, here the Group x Pre/Post interaction was marginally significant ($F = 3.44$, $p < .06$). Further analysis revealed that this interaction was due to the fact that the difference in scores between the experimental group and the control group was greater on the post-test than on the pre-test.

A similar trend emerged in the comparison of the pre- and post-test scores obtained by each group on the part of the test that looked only at the more process oriented problems. However, here the increase in scores from pre-to post-test is too small to reach statistical significance (for exact scores, see Table 2).

The Oral Problem Solving Task

The results from the oral problem solving task revealed no significant differences between the two groups on either the pre-test or the post-test. Moreover, changes from the pre-test to the post-test were only minimal for both groups. For example, the number of children giving the correct answer to the problem increased by only two in the case of the experimental students and by three among the controls. Similarly, the number of students attempting to use a strategy of any kind increased by only two for the experimentals and by one for the controls. The attempt to use more than one mathematical operation to solve the problem was the only variable that was used by noticeably more students on the post-test than on the pre-test, but here the increase occured primarily within the control group (pre-test = 6; post-test = 16). Finally, the only step in the sequence of problem solving activities that was reported as being used by over 50% of the students was "reading" or "trying to understand" the problem. Here, too, the controls improved their performance (pre-test = 54%; post-test = 63%) more than the experimental students, pre-test =56%; post-test = 60%).

The responses to the questions relating to whether students found the problems easy or difficult and whether they could think

of another solution [illegible] analyzed statistically because these responses were found to be unre[illegible] Stu[illegible] for instance, answer that the problem was easy even though they may not have found any way to solve it. Similarly, responses to the question: "Do you know of any other way to solve this problem?" had to be discarded because students were found to answer "yes", but when asked what they would do, they would come up with the same solution they had used before.

The Grade Equivalent Scores

Grade equivalent scores as well as maximum - minimum ranges are given in Tables 4 - 7. The grade equivalents show that, at the time the reading test was administered, i.e., in February, the average scores on that test were only 5! (experimentals) and 48 (controls). The grade equivalents for the mathematics test were somewhat better (experimentals, concepts = 55, problems = 53; controls, concepts = 54, problems = 51). Moreover, the previously noted improvement from pre- to post-test is reflected here, too, particularly for the experimental group on problem solving (experimentals = 57; controls = 55).

The Teachers' Questionnaire

All the teachers indicated that they had enjoyed the entire

teaching experience very much; in fact 50% gave it a rating of 5 on the 5-point scale given in the questionnaire. The teachers were particularly pleased with the workshops which they described not only as useful but also as challenging and motivating. Moreover, the teachers felt that the guidance provided by the workshops was extensive enough to ensure that the module was properly implemented in the classroom.

Details of Implementation

The frequency with which the module was implemented in the classroom varied considerably. One third of the teachers taught the material three times a week during the entire twelve-week period. The others taught it either two or three times a week. One of these teachers taught it more frequently during the first four weeks (four times weekly) and then decreased the intensity during the following weeks. Another teacher taught it only once a week during the first eight weeks and then had daily sessions during the last four weeks.

Fifty percent of the teachers implemented the module during a time specifically set aside for that purpose, the others taught it during the math lessons. In addition, most of them indicated

that they referred to the strategy approach, which was the central feature of the module, during other lessons if the opportunity to do so arose.

Student Reactions

All the teachers reported that their students had loved these special sessions (student reactions to the module were rated between 4 and 5 on the five-point scale).

In order to make the work more exciting for the students, special notebooks were used during the formal sessions. Moreover, much emphasis was put on group discussions, an activity which seemed to promote student participation as the course progressed. Indeed, two thirds of the teachers reported that student participation in group discussions increased from an average of 16% at the outset to 54% towards the end of the twelve-week period. In some classes, student participation was even estimated to be as high as 70% during the latter part of the course.

In spite of the students' positive reactions to the module, the teachers expressed some reservations when asked whether they thought their students were applying what they had learned during the special sessions. Half of the teachers felt that 70% - 80%

of their students applied their newly gained knowledge during their math lessons. The other teachers estimated that fewer than 50% of the students were making any transfer between what was being learned here and their math work. Transfer to other subject areas was considered to be even lower (24% on the average).

SUMMARY AND DISCUSSION

The findings of this study can be summarized as follows:

1. The teachers' attitude to the new teaching approach was very positive
2. Teacher training was reported to have been adequate
3. Student involvement during the classroom sessions was very high
4. Student performance on the problem solving test improved significantly between the pre-test and the post-test. The gains made by the students in the experimental group were more pronounced than those made by the control group
5. On the other written mathematics test, the improvement between the pre-and post-test scores was only marginally significant
6. There were no significant differences between the experimental group and the control group or between the pre and post-test responses on the oral problem solving task.

The most noteworthy outcome of this project is doubtlessly the positive reaction of both teachers and students to this new instructional approach to problem solving. A major contributor to the dynamic classroom setting was the design of the first workshop which had not only given the teachers guidelines on how to implement the module, but had stimulated their own interest in the topic to be taught.

The results from the evaluation also indicate that during the time which elapsed between the pre-tests and the post-tests, the students' ability to solve problems improved more than their familiarity with mathematical concepts. However, this outcome is not wholly attributable to the effect of the module since the control groups' performance on this test also improved signif-icantly, albeit to a lesser degree. Thus, both maturational factors and the effect of the module contributed to this outcome. It should be noted here that in terms of statistical significance, the difference in performance between the experimental students and the controls on the problem solving post-test was only marginal. Moreover, there were practically no differences between the pre- and post-test results on the oral problem solving task. Thus, it would seem that the module had only a limited effect on the development of the students' problem solving skills.

Several factors may have restricted the efficacy of the module. First of all, it should be borne in mind that the module was implemented only during a twelve-week period. During that time, both winter break and Easter holidays occurred so that the actual teaching period was reduced to approximately ten weeks. It is possible that even though there had been, according to the teachers, enough time to teach the material, there had not been enough time for the students to internalize it thoroughly.

Secondly, the module may have had only a limited effect on the development of the specific mathematics skills measured by the CTBS tests. This may be due to the fact that the module focused on the exploration of ways to solve problems whereas the test looked only at the end product, i.e., performance in terms of correct versus incorrect answers to problems. It is true that this issue was addressed through the questions in the oral problem solving task. These questions attempted to elicit from the students information about the strategies they used to arrive at the solution of a problem. However, students may not always be aware of the strategies they use or may not have the verbal means to explain how they got to the solution. It was noted, for instance, that students who had found the correct answer to the problem given in the oral test mostly responded with "I worked it out in my head" when asked what they had done to find the answer.

Finally, many of the students may simply not have applied their recent experience to a novel situation. As noted earlier, the teachers had already expressed their doubts as to whether students would be able to transfer their newly gained knowledge to other problem solving tasks. Here, the degree of internalization may be a key factor. Skills which have not yet been fully mastered may not be called upon even when the opportunity for their use presents itself.

The above discussion suggests that the module needs to be implemented in the classroom for a much longer period of time before its effectiveness can be truly assessed. Moreover, it may be beneficial to show students from time to time the relevance of what is learned here to the solving of problems in the context of their math lessons. In this way, students may be encouraged to try and work out their math problems on basis of a reasoned rather than a mechanical approach. In fact, it may be advisable to implement a suggestion made by some of the teachers, namely, that the module be taught as part of the regular mathematics program during the entire academic year. In view of the enthusiasm generated by this year's experience, such an extension of the module should be well received in the schools.

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Table 1

Sample Sizes

Experimental group	100
Control group	104

Sample Sizes for Individual Tests

Experimental group	48
Control group	46

Participating Schools

Bancroft	Bedford	Guy Drummond
Barclay	Coronation	Sinclair Laird
	Woodland	

Table 2

Summary of Mathematics Test Results[a]

	Pre-Test		Post-Test		F ratios	
	Exp'tal	Control	Exp'tal	Control	Pre/Post	Group
Math Concepts (37)[b]	20.47	20.33	21.07	20.74	2.76*	0.99
Problems (27)[c]	15.40	15.38	17.00	16.00	15.41***	0.01
Special Problems (7)	3.54	3.41	3.84	3.46	1.89	1.30

[a] Means shown are adjusted according to results on Raven Matrices
Unadjusted means are shown in Table 4

[b] Maximum scores in parenthesis

[c] Marginally significant Pre/Post x Group interaction $F=3.44$ $p \leq .06$

* $p \leq .05$
*** $p \leq .001$

Table 3

Summary of Results from the Oral Problem Solving Task

	Pre-Test					Post-Test				
	Exp'tal		Control			Exp'tal		Control		
	No	%	No	%	x^2	No	%	No	%	x^2
Correct answers	3	6	1	2	0.22	5	10	4	9	0.00
Read the problem	27	56	25	54	0.00	29	60	29	63	0.00
Used strategies	6	13	4	11	0.07	8	17	5	11	0.26
Used more than one operation	10	21	6	13	0.53	12	25	16	35	0.66
Checked back	3	.6	1	2	0.22	5	10	4	9	0.00

Table 4

Unadjusted Means and Grade Equivalents

PRE-TEST/EXPERIMENTAL GROUP

	Means	SD[a]	Maximum	Minimum	GE[b]
Reading (54)[c]	26.25	9.29	47	6	51
Math Concepts (37)	20.37	6.62	35	6	55
Problems (27)	15.32	4.81	25	2	53
Special problems (7)	3.54	1.78	7	0	—

[a] SD = Standard Deviation

[b] GE = Grade Equivalent

[c] Maximum scores in parentheses

Table 5

Unadjusted Means and Grade Equivalents

PRE-TEST/CONTROL GROUP

	Means	SD[a]	Maximum	Minimum	GE[b]
Reading (54)[c]	24.75	8.31	50	10	48
Math Concepts (37)	20.26	5.58	35	6	54
Problems (27)	15.29	4.79	25	4	51
Special Problems (7)	3.41	1.78	7	0	—

[a] SD = Standard Deviation

[b] GE = Grade Equivalent

[c] Maximum scores in parentheses

Table 6

Unadjusted Means and Grade Equivalents

POST-TEST/EXPERIMENTAL GROUP

	Means	SD[a]	Maximum	Minimum	GE[b]
Reading (54)[c]	NA	—	—	—	—
Math Concepts (37)	21.04	6.33	36	4	57
Problems (27)	16.92	5.68	27	6	57
Special Problems (7)	3.64	1.90	7	0	—

NA - not administered

[a] SD - Standard Deviation

[b] GE - Grade Equivalent

[c] Maximum scores in parentheses

Table 7

Unadjusted Means and Grade Equivalents

POST-TEST/CONTROL GROUP

	Means	SD[a]	Maximum	Minimum	GE[b]
Reading (54)[c]	NA	—	—	—	—
Math Concepts (37)	20.75	6.71	35	9	57
Problems (27)	15.96	5.43	26	5	55
Special Problems (7)	3.46	1.83	7	0	—

NA - not administered

[a] SD - Standard Deviation

[b] GE - Grade Equivalent

[c] Maximum scores in parenthesis

Appendix I

Explanation of Analysis of Covariance

Analysis of covariance is a statistical procedure which is similar to the analysis of variance in that it is used to compare the results of two or more groups on a particular test. At the same time, analysis of covariance mathematically reduces the influence that another variable, such as I.Q., might have on the test results. For example, if two hypothetical groups under investigation differ *a priori* in terms of I.Q., and if it is believed that I.Q., will significantly influence performance on the test being administered (e.g., a reading test), then an analysis of covariance will estimate the significance of the reading factor. This mathematical procedure is used when it is difficult or impossible to select groups which are equivalent in I.Q.